

Controlling a leaky tap

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Abstract

We apply the Ott, Grebogy and Yorke mechanism for the control of chaos to the analytical oscillator model of a leaky tap obtaining good results. We exhibit the robustness of the control against both dynamical noise and measurement noise. A possible way of controlling experimentally a leaky tap using magnetic-field-produced variations in the viscosity of a magnetorheological fluid is suggested.

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The realization that the majority of natural phenomena are chaotic led to the suggestion of chaotic behavior in the common household phenomena of a leaky tap or dripping faucet [1]—common perhaps, but not well understood, not even the process of drop formation where the flow changes from a fluid mass to one or several falling drops; see, for example, [2–4] and the references therein. The first clear experimental evidence of such chaotic behavior was found by Shaw and his collaborators [5,6], further evidence was found a few years later by Wu and Schelly [7] and by Núñez-Yépez *et al.* [8]. Since then many experiments and theoretical works have established the system as a sort of paradigm for dissipative chaos [9–16].

Shaw proposed the first model for the process, a variable mass oscillator inspired in Rayleigh ideas [17, 18]. The model was actualized by Sánchez-Ortiz and Salas-Brito (SOSB in what follows) [19, 20] and independently by D’Innocenzo and Renna [21–24] which, by changing the breakup mechanism and the way of choosing initial conditions, showed the broad range of behavior that can be achieved using the model and how it can be qualitatively related to the experimental facts. A promising hydrodynamic model, aiming at a *quantitative* agreement with the experiment and accounting for some of the topology transitions and the singularities in the phenomenon, has been recently put forward [25]. It must be clear that despite the enormous simplification in reducing a many-degrees-of-freedom fluid system to an one-dimensional model, there are many things that can be understood using the oscillator model since, basically due to dissipation, the system restricts itself to essentially one-dimensional attractors. Kiyono, Ishioka and Fuchikami have actually shown that the agreement between a model and the experimental results can be made quantitative by analysing the system from the perspective of fluid mechanics [25]. Using such ideas, Kiyono and Fuchikami have improved the relaxation oscillator model [26]. The oscillator idea illustrates the important and sometimes underemphasized point that for reproducing qualitative and even quantitative features of a chaotic system it is usually not necessary to use very complex models.

Moreover, the leaky tap and the oscillator model have been used as a sort of role model to simulate other complex phenomena [27]; furthermore, given the similarity between certain of their features [28], they can be of help in modelling the comparable-to-chaotic heartbeat behavior [28–30]. The experimental control of a leaky tap can then be of importance as a testing ground for certain ideas. For instance, a cardiac MR imaging technique has been proposed which employs time series forecast and standard methods for the analysis of chaos on heartbeat

time series. Using such concepts, new pacemakers are being investigated which use control techniques to correct arrhythmic behavior of the heart while minimizing their intervention and battery consumption. See [31] and the references therein.

Our aim in this work is to apply a model independent chaos control technique to the SOSB equations in the analytical approach of D’Innocenzo and Renna [24]; we then suggest an experimentally-realizable scheme for the control of an actual dripping faucet. We carry out the control using the Ott, Grebogy and Yorke strategy (OGY in what follows) [32]; the advantages of the OGY method is that it does not need a detailed knowledge or model of the phenomena and it uses the chaotic behavior itself as the mechanism of control. We have found that it is possible to stabilize the SOSB model around one of its unstable equilibrium points and that such control is robust (between certain limits) against external perturbations; this is obviously a good feature with an experiment in mind. The control is accomplished by adjusting the parameter of the SOSB model analogous to the viscosity of the leaking fluid.

Let us begin by reviewing the SOSB relaxation oscillator model [33,34]. The starting equations, in nondimensional coordinates, are [19,20]

$$\begin{aligned}\frac{dx}{dt} &= y, \\ \frac{dy}{dt} &= -\frac{1}{m}(x + \beta y) + g, \\ \frac{dm}{dt} &= f,\end{aligned}\tag{1}$$

where, β , g and f are parameters modelling viscosity, external force (gravity) and water inflow, respectively, and m is the mass.

We here, following the analytic approach of D’Innocenzo and Renna [23] and instead of studying directly numerical solutions to the set of equations (1), use a sort of approximate solution to it, namely

$$x(t) = [A \sin \omega(t) t + B \cos \omega(t) t] \exp(-\gamma(t) t) + m(t) g,\tag{2}$$

where $m(t) = m_0 + f(t - t_0)$, $\gamma(t) \equiv \beta/m(t)$ and $\omega^2(t) \equiv 1/m(t)$; equation (2) together with the following proviso for the drop breakup: when the position of the oscillator reaches the meniscus length (normalized such that $x_c = 1$) a drop is forced to detach, provoking an abrupt diminution in the oscillator mass by the quantity [19,20]

$$\Delta m = h m(t_c) y(t_c), \quad (3)$$

h being a model parameter and where we have set $t \equiv t_c$. So before using (2) again, we have to reset the starting value of the oscillator mass to the new value $m_0 = m(t_c) - \Delta m$; this scheme substitutes for the missing singularity-forming description of drop detachment [19–21]. The analytic model (2) was first proposed by D’Innocenzo and Renna [21] and has been recently used to reproduce [24] the closed loop attractors and the Hopf bifurcations experimentally observed in a leaky tap [7, 15–16].

We further assume that the clock resets every time a drop breaks off, *i.e.* we take $t_0 = 0$ for every new drop. To give a criterion for the initial position of the next drop we use a sort of amorphous drop model, loosely inspired in Eggers study [2]. This has lead us to propose a way of choosing the initial conditions after breakup such that

$$x_0 = \exp(-\Delta m), \quad (4)$$

this amorphous drop mechanism guarantees that $0 < x_0 < 1$. Furthermore, the initial velocity of the remaining fluid can always be taken as $y_0 = y(t_c)$ simply considering that the mass of the remaining fluid, *plus* the effect of the mass inflow, produce an effective mass term very large compared to Δm on which the snapping back of the system has a negligible effect. The use of the amorphous model (4) had, in fact, its origin in our attempts at using the spherical drop model of D’Innocenzo and Renna [21–23] that, sometimes, resulted in detached drops much larger than the normalized meniscus lenght.

Our analytical SOSB model is hence specified by equations (2) and (3) together with the amorphous drop scheme for the initial position of the next drop. Such mechanism of releasing drops, besides giving reasonable values for the initial positions generates important correlations between successive drops.

Although it is known that the results obtained from the model vary greatly according to the specific mechanism employed for simulating the drop detachment [23], we here limit ourselves to quote results from the amorphous drop mechanism (4). We point out that we have tried different mechanisms and other variations of the SOSB model getting the same general conclusion [33].

Using a modified Newton-Raphson method [35] for getting t_c from the dripping condition $x(t_c) = 1$, a very efficient method of simulating the dripping tap behavior

is obtained [21,22]. We should point out that the analytical SOSB equations (2) and (3) are appropriate for modelling leaking from relatively big (with diameters larger than $\sim 1/4$ cm) faucets, since for such diameters the drop dynamics is mainly governed by the center of mass motion of the hanging fluid [2, 17]. We are then capable of accumulating a large number of drip intervals $t_n \equiv t_c^{(n)}$ for analysis. The drip intervals, *i.e.* the time spans separating a drop from the next one, have become the standard variables used in all leaky tap studies.

Bifurcation diagrams—dripping spectra in the terminology of Wu and Schelly [7]—, time series and return maps t_{n+1} versus t_n , illustrating the results that can be obtained from the analytical SOSB equations, are shown in Figure 1 and 2 [33, 34]. Figure 1 shows both a bifurcation diagram (figure 1a) and a time series (figure 1b) taken from the zone we want to control. Notice also that at the parameter values ($g = 0.4$, $h = 0.3$ and $\beta = 3.01$) used the system undergoes a period-doubling sequence, shows evidence of crisis (figure 1a) and behaves intermittency (figure 1b) [18]. The horizontal dashed line in figure 1b simply marks the drip-interval at the unstable period-1 point ($t_c = t_F = 0.2858$) we aim to control (but it does **not** necessarily coincide with the seemingly intermittent state appearing in figure 1b). The bifurcation diagram also shows that, in the conditions of figure 1a, dripping is interrupted by “continuous flow” at $f > 8.5002$, that is, at greater f -values the oscillator position always remains larger than the meniscus length after the detachment of the first drop [19,20].

Let us mention that the parameter values used in our discussion do not attempt to be typical of an experimental situation, rather they were chosen with the sole purpose of illustrating the OGY mechanism as applied to the system. We must mention though that the results were checked for other values of the parameters, that is, for chaotic attractors of different sorts, always obtaining similarly good results—although the dynamics may be different but still chaotic. The method even allowed us to control the system in an unstable period-10 cycle [33].

Figure 2 shows the reconstructed attractor that exists at the unstable fixed point location. The fixed point is shown as a small black dot in the figure 2 inset. Notice that the reconstructed attractor has a complex structure [20] which can be regarded as difficulting the precise location, and hence the control, of the unstable fixed point. Nevertheless, the location of such fixed point and its control are easily achieved and are basically limited by the precision of our computations.

To identify the unstable fixed point orbit (shown in Figure 2) in the otherwise chaotic attractor-dominated dynamics, we simply acknowledge that every chaotic

system has an agglomeration of unstable periodic orbits embedded almost anywhere. Hence, there must be a fixed point whenever an attractor crosses the line of the identity ($t_{n+1} = t_n$) in a return map. Using the time series, we can numerically identify the unstable fixed point at $t_c = t_F = 0.2858$ present in the attractor shown in figure 2.

Around the unstable fixed point $\mathbf{X}_F = (t_n = t_F, t_{n+1} = t_F)$ in the return map, with the help of the time series, we can use a locally linear dynamics [32] to describe the system

$$D \cdot (\mathbf{X}_n - \mathbf{X}_F) = \mathbf{X}_{n+1} - \mathbf{X}_F, \quad (5)$$

where D is a 2×2 matrix and \mathbf{X}_n is the vector with components (t_{n+1}, t_n) . With the local dynamics (5), it is then a simple matter to calculate the normalized D -eigenvectors, \mathbf{e}_s , \mathbf{e}_u , and its corresponding eigenvalues, λ_s , λ_u , associated with its stable and unstable manifolds. From them, we can evaluate also the contravariant vector associated with the unstable manifold as the vector \mathbf{f}_u , for which $\mathbf{f}_u \cdot \mathbf{e}_u = 1$ and $\mathbf{f}_u \cdot \mathbf{e}_s = 0$ holds [32].

To control the system we have chosen to adjust the viscosity parameter β ; we choose β and not the seemingly more natural fluid inflow, because we have in mind a magnetorheological fluid in which the viscosity can be varied using an easily tuned magnetic field and more important because it is rather difficult to control f with confidence, in our conditions at least. We thence need to evaluate—numerically from the time series—the sensitivity of the model to changes in the viscosity parameter β respect to a fiducial value β_0 , as

$$\mathbf{s} = \left. \frac{\partial \mathbf{X}_F}{\partial \beta} \right|_{\beta_0}. \quad (6)$$

With the above information we let the system run and apply the control every time the drip interval is within an appropriate fixed-point neighborhood; such neighborhood is specified through the inequality

$$|(\mathbf{X}_n - \mathbf{X}_F) \cdot \mathbf{f}_u| < \xi_*, \quad (7)$$

where

$$\xi_* = \left| \delta \beta_* (\mathbf{s} \cdot \mathbf{f}_u) \left(1 - \frac{1}{\lambda_u} \right) \right|, \quad (8)$$

and $\delta\beta_*$ is the maximum value allowed (see below for the value set) for changes in the viscosity parameter—we pinpoint that (8) is only valid as a first-order approximation. Once with the control working and after a brief transitory the system never gets far from \mathbf{X}_F , as figures 3a and 3b show. Notice that, again, we choose controlling the dynamics around the interval defined by (7) and (8); this is quite appropriate with an experimental situation in mind. The scheme described is simply the OGY control method, forcing the system to evolve towards the unstable direction by changing slightly the value of the β parameter [32].

The OGY scheme, applied to the model dynamics lead to the results shown in Figure 3. The results correspond to an unstable fixed point $t_F = 0.2858$, found within the chaotic attractor at the parameter values $f = 8.49$, $\beta = 3.01$, $g = 0.4$ and $h = 0.3$; the behavior is intermittent at these parameter values (figure 1b). The control parameters used are $\delta\beta_* = 1.3$, $\xi_* = 0.008$, $\mathbf{s} = (-0.0021, -0.0021)$, and the unstable eigenvalue is $\lambda_u = -1.24$; the unstable manifold is associated with the contravariant vector

$$\mathbf{f}_u = \begin{pmatrix} -0.8940 \\ 0.7020 \end{pmatrix}. \quad (9)$$

The explicit expressions given above for the unstable and stable eigenvectors guarantee that we are not in an homoclinic tangency point of the attractor [36] which is an unsuitable point for applying the OGY method. In Figure 3a we can observe a consistent estabilization of the system after the application of the control in $n = 1000$; it takes less than 150 drippings to get the system into the fixed point. The control is released after drop 3000 and chaos sets in immediately; it is again applied at $n = 5000$, and 150 or so drippings ahead the system becomes periodic again. Further information about the approach to the fixed point once the control is applied, can be obtained from a return map of the process; this is shown in Figure 3b. The spiralling approach, clearly shown in the inset on figure 3b, to the fixed point seems to be typical. We have to conclude then that with no perturbations present the control seems to work very well.

An appropriate question is what happens if there are extra random perturbations. Such perturbations are expected to occur in any experimental realization of the leaky tap. In what follows we first analyse the effect of random noise superimposed to the value of the parameter f . We should term this the case of dynamical noise, since experimentally it arises from the impossibility of keeping perfectly fixed the inflow. In fact, we choose such parameter to illustrate the robustness of the control precisely because the fluid flow into the tap is a difficult

variable to keep fixed in an experimental situation [8, 12]. This also explains why we do not considered proposing an experimental mechanism of control using the inflow f —though the control is equally easy to achieve adjusting f but *in the model* [33]. The random perturbation is applied as $f = f_0 + \delta f$ in the model equations, where f_0 is now the fiducial value (*i.e.* $f_0 = 8.49$, as in figures 2 and 3) and δf is a uniformly distributed random variable in $[-0.0043, 0.0043]$. We have to be sure that such random perturbation does not significantly change the dynamics since the f -width of the chaotic zone is small. In figure 4 we show, as an example, a time series and a return map with fiducial values of the parameters as in Figure 2, but with the random perturbations applied to f allowing for variations up to 1% of its fiducial value (note that such variations represent 10% of the total width of the chaotic zone). It can be seen that the dynamics just become fuzzier compared to the original unperturbed case. Notice also that the system does not permit imposing larger variations in f , otherwise we will leave the rather small chaotic zone (the f -width of that zone is 0.035, as shown in Figure 1) and the dynamics would then be drastically changed.

What happens with the control scheme turned on? The control was applied without modification to the perturbed system and the results show that the scheme is rather robust under random perturbations in f . Figure 5 show time series and return maps of the randomly perturbed system under control. Notice that the spiral approach to the fixed point has become an ellipsoidal blob of points; this figure roughly corresponds to the area of the interval (8) in the reconstruction space. Incidentally, notice that figures 4b and 5b also illustrates the predicted noise induced attractor deformation and elongation [36] recently observed in periodically driven non-linear electric circuits [37]. Figure 5b also illustrates that the control effectively stabilizes the system to a neighborhood of the fixed point, not allowing vagaries larger than the maximum size of the control zone.

But the lack of control in f is not the only perturbation worth of analysing. The unavoidable uncertainties in the time measurements, that is, what we can term the case of measurement noise, and the problem of lost drops are also important. We simulate such behavior by randomly perturbing the values of the drip intervals calculated from the model. We consider thus drip intervals $t_n = t_n^{(0)} + \delta t_n$ where $t_n^{(0)}$ is the drip interval calculated from (2) and (3) and δt_n is a random variable with, again, a uniform distribution. What we found using these ‘measured’ drip intervals is that, if the uncertainty introduced by the random noise is larger than $\sim 0.5\%$ of the maximum value of the drip interval, despite the intended control,

the system exhibits little but noticeable chaotic bursts. The bursts become larger as the uncertainty grows until the control is completely lost. This is shown for successively larger values of the perturbation in Figures 6a, 6b and 6c. In the last of the time series shown (Fig. 6c, with a perturbation of $\simeq 10\%$ of the total t -width ($\simeq 0.02995$) of the chaotic zone) traces of the control still are noticeable but overall the system is destabilized and chaotic. Such behavior can be easily understood when it is considered that at such uncertainties it is no longer possible to tell apart a drop from the adjacent ones. In this measurement noise case then, it is possible to quote the noise values which the control mechanism found acceptable, whereas in the previous dynamical noise case it was not possible due to the small f -width of the chaotic zone [7, 12, 15]; in the dynamical noise case the system would no longer be within the chaotic zone before the control collapses by increasing the noise level. That dynamical noise could throw the system out from the chaotic regime, can also happen in the experiment [8] but, in such a case, the large fluctuations in f would simply mean that the experiment is not working properly.

In all the examples given, the control procedure used is applied using the approximate linear dynamics calculated from the unperturbed system, which is a sort of idealistic case. In a more realistic situation, the local dynamics will be evaluated from the actual measurements and this would improve the control.

The results of our computations with the relaxation oscillator SOSB equations hint towards a control technique applicable to the leaky tap in an experimental situation. We require a system with at least a parameter allowing quick adjustment and quick response times as compared with typical drip intervals; typical values of t_n in an experiment are of the order of 100 ms [8, 12, 16]. The chosen control variable should allow faster response than this typical value. We have thought therefore on adjusting the fluid viscosity because the inflow f is not easy to control, at least from the viewpoint of our Laboratory. On the other hand, common fluids (water is the working fluid in every experiment performed to date) are very difficult to change their viscosity excepting with changes in temperature, but this is not easy to accomplish in the required circumstances. Had we thought of changing the temperature of the water, we would need rather large changes which would also change other system parameters—as the diameter of the nozzle—and temperature would not be so easy to control.

To overcome such anticipated difficulties, we propose the use, instead of the customary water, of an oil-based magnetorheological fluid as the leaking fluids in the system. Such fluids are easy to obtain, have response times of 2 or 3 milliseconds

—almost an order of magnitude below the typical drip intervals in water and the drip intervals are larger in the magnetorheological fluid given its greater viscosity. Besides, they can quickly change their viscosity for up to a 10^6 factor [35] (though for our purposes we do not need such huge changes) simply applying a magnetic field, which is also rather easy to adjust. Giving such characteristics, we think that the method would allow an excellent control.

In summary, we have applied successfully the OGY control method to the SOSB leaky tap model, investigating the possible ill-effects of random noise on the water inflow into the tap and on the drip intervals. We have found the the control procedure is effective up to noise to signal ratios of the order of $\sim 10\%$. We should also mention that all the computations reported in this article were carried out in fortran 77 using a PC workstation running under Linux.

To finalize, we have to say that the study in [25] has been further used to improve the oscillator model. The main change has been the use of a mass-dependent elastic ‘constant’ k for the spring [26] (which we here normalized to 1); with the proper identification of the model parameters a very good agreement with the experimental values [6–10] is found. This adds to the usefulness of the oscillator model as it is further illustrated by this contribution.

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Figure Captions.

Figure 1.

Illustration of the behavior predicted by the relaxation oscillator model for the leaky tap. Many other examples of the possible behavior can be found in [21–25, 33,34].

1a. Bifurcation diagram at $\beta = 3.01$, $g = 0.4$, $h = 0.3$ as f is varied. Notice that beyond $f = 8.500019$ the dripping stops and “continuous flow” sets in. The vertical dashed line passing through $f = 8.49$ marks the zone to control. Notice that the chaotic zone after the period doubling bifurcations extends roughly from 8.465 to 8.500, a total f -width of 0.035.

1b. Time series in the zone we want to control ($\beta = 3.01$, $g = 0.4$, $h = 0.3$ and $f = 8.49$). Notice the signals of intermittency. The thin dashed line $t = 0.2858$ corresponds to the unstable fixed point we intend to stabilize. Let us emphasize that we do not intend to control the intermitent orbit which seems to coincide with the selected unstable fixed point.

Figure 2.

Return map t_{n+1} vs. t_n showing the unstable fixed point at $t_F = 0.2858$. The inset is a blow up of the square neighborhood depicted around the fixed point. Such unstable orbit is pointed to by a black arrows and marked by a black dot in the inset. As you can notice, the attractor has a complex structure composed of at least two very close sheets. The fixed point lays in the innermost sheet of the reconstructed attractor.

Figure 3.

Effect of the OGY scheme on the dynamics; compare with figures 1b and 2.

3a. Time series of drip intervals for the unperturbed model with the control turned on and off. At $n = 1000$ the control is applied, it takes roughly 150 drops for the system to be stabilized into the unstable fixed point at $t = t_F = 0.2858$. At $n = 3000$ the control is released and chaotic behavior sets in immediately. At $t = 5000$ the control is applied again.

3b. Return map of the control process. Notice the spiral approach to the unstable fixed point when the control is turned on. The inset is a blow up of the region, exactly the same as described in figure 2, around the unstable fixed point.

Figure 4.

The SOSB model in the presence of random perturbations applied to the value of f . The noise level is $\simeq 10\%$ of the f -width of the chaotic zone. In this case, it is not possible to increase the noise for testing the robustness of the control without

first leaving the rather small (f -width = 0.035) chaotic zone.

4a. Time series of the drip intervals with random noise superposed on f . A comparison with figure 1b may show that the dynamics gets fuzzier.

4b. Return map of the zone to be controlled with random noise on f superposed. Compare to figure 2. Notice the deformation and the elongation of some parts of the reconstructed attractor induced by the applied random noise [36,37]. The fuzziness mentioned in 4a becomes evident.

Figure 5.

The OGY scheme applied to the SOSB model in presence of random noise on f . The noise level is $\simeq 10\%$ of the f -width of the chaotic zone.

5a Time series of the f -perturbed SOSB leaky tap model, with the control turned on at $n = 5000$. Despite the noise the system stabilizes around the unstable fixed point.

5b. Return map of the system with the control turned on. The spiral approach to the fixed point becomes an approximately elliptical region where the system gets controlled.

Figure 6.

Effect of random noise applied to the drip intervals. Notice that in the conditions of figure 6b it begins to be difficult to tell apart a drop from adjacent ones and that, in the conditions of figure 6c, it is almost not possible.

6a. The noise level is here 0.5% of the maximum range allowed for t . Control is still rather good.

6b. The noise level is 0.75% of the maximum range in t . The bursts of chaos where control is lost are evident, control is also present though far from perfect.

6d. The noise level is 1% of the maximum range in t . Traces of control still remain but it is almost completely lost.

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